Statistical Time Series Methods for Vibration–Based SHM

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Abstract

A concise overview of the principles of statistical time series methods for vibration–based Structural Health Monitoring is presented. The structure of the methods is outlined and their basic features and operation are explained. The use of various methods, scalar or vector, non–parametric or parametric, is then demonstrated via their application to damage diagnosis on a laboratory–scale aircraft skeleton structure.

1 INTRODUCTION

Statistical time series methods for vibration–based Structural Health Monitoring (SHM) utilize random excitation and/or vibration response signals (time series records), along with statistical model building and decision making tools, for inferring the health state of a structure. They form an important and powerful category under the so–called *data–based* methods, which are themselves part of the broader *vibration–based* family [1, 2, 3, 4, 5, 6, 7, 8, 9].

As such, time series methods share the advantages of the broad vibration–based family (for example by being "global", in the sense that no measurements near a damage location are needed for effective SHM), as well as those of the data–based methods (for example by circumventing the need for detailed physics–based models of the structural dynamics, like Finite Element type models or modal models). Moreover, the statistical nature of the methods offers effective treatment of uncertainties in the signals employed, plus in the statistical decision making which may be set–up to operate with specif ed performance characteristics [7, 8].

Statistical time series methods for SHM utilize *scalar* or *vector* random (stochastic) vibration signals under healthy and potentially damaged states, identif cation of parametric or non–parametric time series models of the dynamics of each each state, and extraction of a statistical characteristic quantity Q characterizing the structural state in each case (*baseline phase*). *Damage diagnosis* (the term signif es the collection of all subproblems involved in SHM, namely damage detection, identif cation, and magnitude estimation) is then accomplished via statistical decision making and estimation techniques implemented in the *inspection phase*. In this the current characteristic quantity, say Q_u, is "compared" (in a statistical sense) to that of each potential state as determined in the baseline phase (for example Q_o of the healthy state for damage detection). For an extended overview of the principles and techniques of statistical time series methods for vibration–based SHM the interested reader is also referred to [7, 8, 10].

Non–parametric time series methods are those based on scalar or vector non–parametric time series representations, such as the Power Spectral Density (PSD) of the Frequency Response Function (FRF) [7, 8], and have received considerable attention in the literature [11, 12, 13]. *Parametric* time series methods are those based on scalar or vector parametric time series representations, such as the AutoRegressive Moving Average (ARMA) models [14, 7, 8]. This latter category has attracted signif cant attention in recent years [15, 16, 17, 18].

The *goal* of this article is to provide a concise overview of the principles and techniques of statistical time series methods for vibration–based SHM and demonstrate their application on a laboratory structure. The effectiveness of certain scalar (univariate) and vector (multivariate) methods, of both the non–parametric and parametric types, is also assessed. It should be stressed that the article aims, primarily, as a concise introduction to the topic. As such it is restricted to the presentation of some of the simplest methods – the interested reader is directed to the literature for additional and more elaborate methods. For instance see the books [3, 9], the book chapters [8, 10], the article [18] on an assessment of various methods, the articles [19, 20] on advanced methods allowing for precise damage localization, and the article [21] on damage diagnosis in Time-Varying structural systems.

The rest of this article is organized as follows: The SHM problem is described in section 2, and the structure of statistical time series methods for SHM is reviewed in section 3. A concise overview of certain statistical time series methods for vibration–based SHM is provided in section 4. The scale aircraft skeleton structure used in this study and the experimental set–up are presented in Section 5, while SHM results are presented in Section 6. The conclusions are f nally summarized in Section 7.

2 THE SHM PROBLEM

Let $\mathcal{S}_o$ designate the vibrating structure of interest in its *nominal* (healthy) state. Also let $\mathcal{S}_A, \mathcal{S}_B$, ... designate the structure under *damage types* (*damage modes*) A,B,... respectively. Each damage type may – of course – include a continuum of damage sizes (magnitudes), all characterized by common nature and/or location (for instance damage of various possible magnitudes to a specif c structural element). The structure under a specif c damage, say of type A and magnitude k, is designated as $\mathcal{S}_A^k$. The symbol F_A^k is also used for designating the damage itself.

During inspection (which may be continuous or periodic in time) the structure is supposed to be in an unknown health state $\mathcal{S}_u$, which needs to be determined based on fresh vibration signals. In general these may include the force excitation $x_u[t]$ and/or vibration response $y_u[t]$ $(t = 1, 2, \ldots, N)$ signals (t designates discrete time, with the corresponding analog time being $(t-1) \cdot T_s$, with T_s standing for the sampling period; the subscript "u" designates the current/unknown structural health state).

Let the complete signal records obtained during inspection be designated as $(x_u)_1^N$ and $(y_u)_1^N$, and let the excitation–response signals be collected into the vector $\boldsymbol{z}_u[t] = [x_u[t] \; y_u[t]]^T$ $(t = 1, 2, \ldots, N)$ (lower/upper case bold face symbols designate vector/matrix quantities, respectively; by convention all vectors are column vectors). The complete data record is then designated as $(\boldsymbol{z}_u)_1^N$.

It should be noted that all collected signals need to be suitably pre-processed [2, 22]. This may generally include low pass or band pass signal f ltering within a selected bandwidth (frequency range), signal subsampling (in case the originally used sampling frequency is too high), as well as proper scaling. The latter is used for numerical reasons, but also for counteracting – to the extent possible – different operating (including excitation levels) and/or environmental conditions. In the case of linear time–invariant (stationary) structural dynamics, scaling typically involves subtraction of each signal's sample (estimated) mean and normalization by its sample (estimated) standard deviation. In case of multiple excitations care should be exercised in order to ensure minimal crosscorrelation among them. Scaling may *not* be generally applied in the case of non-linear structural or non–stationary dynamics.

Given the data $(\boldsymbol{z}_u)_1^N$, collected during the inspection phase, the problem of SHM (determining the current health state of the structure) may be analyzed into three subproblems:

(a) *Damage detection*, which is the binary decision making subproblem in which the mere presence of damage is determined ($\mathcal{S}_u = \mathcal{S}_o$ or $\mathcal{S}_u \neq \mathcal{S}_o$).

(b) *Damage identif cation*, which is the multiple decision making subproblem pertaining to the identif cation (characterization, localization) of a detected damage. In the present context damage type (mode) A ($\mathcal{S}_A$), B ($\mathcal{S}_B$), and so on.

(c) *Damage estimation* is the subproblem pertaining to damage magnitude (size) estimation.

3 THE STRUCTURE OF STATISTICAL TIME SERIES METHODS

3.1 The Operational Viewpoint

From an *operational viewpoint* the tackling of the aforementioned subproblems requires – in addition to $(z_u)_1^N$ – the availability of similar data records from the nominal (healthy) structure and also from the structure under each damage state. That is, in present terms, $z_o[t]$ $(t = 1, 2, \ldots, N)$ corresponding to $\mathcal{S}_o$, as well as $z_A[t]$,$z_B[t]$, ... $(t = 1, 2, \ldots, N)$ corresponding to $\mathcal{S}_A, \mathcal{S}_B$, ..., respectively. Additionally, data records corresponding to various damage magnitudes within each damage type are required. These data records may be obtained either from the actual structure (whenever possible), or from laboratory scale models, or from detailed simulation models.

The data records are obtained and processed in an initial *baseline phase*. This is done once. On the other hand, the current data acquisition, processing, and decision making are taking place in a second operational phase that is referred to as the *inspection phase*.

3.2 The Conceptual Viewpoint

From a *conceptual viewpoint* statistical time series methods include analysis (modelling) and statistical decision making. (a) The analysis part includes characterization and parametric or non-parametric modelling of part of the dynamics. The aim is the extraction, from each data record, of a characteristic quantity, designated as Q (which is a function of z_1^N) and plays an instrumental role in the decision making part. (b) In the statistical decision making part decisions are made by "comparing", via formal statistical hypothesis testing procedures, the current characteristic quantity Q_u to its counterparts Q_o, Q_A, Q_B, ... pertaining to the various possible structural states (o, A, B, ..., respectively).

Damage detection is then formulated as a binary composite hypothesis testing problem expressed as:

$$\begin{aligned} H_o &: \quad Q_o \sim Q_u && \text{(null hypothesis - healthy structure)} \\ H_1 &: \quad Q_o \not\sim Q_u && \text{(alternative hypothesis - damaged structure)} \end{aligned} \tag{1}$$

with $\sim$ designating a proper relationship (such as equality, inequality, and so on).

Damage identif cation, is formulated as a multiple hypothesis testing problem which may be expressed as:

$$\begin{aligned} H_A &: \quad Q_A \sim Q_u && \text{(hypothesis A - damage type A)} \\ H_D &: \quad Q_B \sim Q_u && \text{(hypothesis B - damage type B)} \\ &\vdots \qquad \vdots \end{aligned} \tag{2}$$

Damage estimation is a generally treated via interval estimation techniques.

Remarks: The design of a binary statistical hypothesis test (such as that of Equation (1)) may be based upon the probabilities of *type I* and *type II* error occurrence. The f rst – designated as α and also referred to as the *type I risk* – is the probability of rejecting the null hypothesis H_o although it is true (*false alarm*). The second probability – designated as β and also referred to as *type II risk* – is the probability of accepting the null hypothesis H_o although it is not true (*missed fault*). The designs treated in this article are based upon selected type I error occurrence probability (α), and utilize the probability density function of a relevant random quantity under the null (H_o) hypothesis of a healthy current structure. In selecting α it should be born in mind that a decrease/increase in it results in a corresponding increase/decrease in β. The reader is referred to references such as Basseville & Nikiforov [23, subsection 4.2] and Montgomery [24, subsection 3.3] for details on statistical hypothesis testing.

Table 1: **Characteristics of statistical time series methods for SHM**

Method	Principle	Test Statistic
PSD based	$S_u(\omega) \stackrel{?}{=} S_o(\omega)$	$F = \widehat{S}_o(\omega)/\widehat{S}_u(\omega) \sim F(2K, 2K)$
FRF based	$\delta\lvert H(j\omega)\rvert = \lvert H_o(j\omega)\rvert - \lvert H_u(j\omega)\rvert \stackrel{?}{=} 0$	$Z = \delta\lvert \widehat{H}(j\omega)\rvert/\sqrt{2}\widehat{\sigma}_H \sim N(0, 2\sigma_H^2(\omega))$
Residual variance	$\sigma_{oo}^2 \stackrel{?}{\geq} \sigma_{ou}^2$	$F = \widehat{\sigma}_{ou}^2/\widehat{\sigma}_{oo}^2 \sim F(N, N-d)$
Model parameter	$\delta\boldsymbol{\theta} = \boldsymbol{\theta}_o - \boldsymbol{\theta}_u \stackrel{?}{=} 0$	$\chi_\theta^2 = \delta\widehat{\boldsymbol{\theta}}^T (2\widehat{\boldsymbol{P}}_\theta)^{-1}\delta\widehat{\boldsymbol{\theta}} \sim \chi^2(d)$
Residual likelihood	$\boldsymbol{\theta}_o \stackrel{?}{=} \boldsymbol{\theta}_u$	$\sum_{t=1}^{N}(\boldsymbol{e}_u^T[t, \boldsymbol{\theta}_o] \cdot \boldsymbol{\Sigma}_o \cdot \boldsymbol{e}_u[t, \boldsymbol{\theta}_o]) \leq l$

$S(\omega)$: Power Spectral Density (PSD) function; $\lvert H(j\omega)\rvert$: Frequency Response Function (FRF) magnitude
σ_H: standard deviation of $\lvert\widehat{H}_o(j\omega)\rvert$; $\boldsymbol{\theta}$: model parameter vector; d: parameter vector dimensionality; $\boldsymbol{P}_\theta$: covariance of $\boldsymbol{\theta}_o$
σ_{oo}^2: variance of residual signal obtained by driving the healthy structure signals through the healthy model
σ_{ou}^2: variance of residual signal obtained by driving the current structure signals through the healthy model
e: k-variate residual sequence; $\boldsymbol{\Sigma}$: residual covariance matrix; l: user def ned threshold; N: signal length in samples
In all cases estimators/estimates are designated by a hat.
The subscripts "o" and "u" designate healthy and current (unknown) structural state, respectively.

3.3 Types of time series methods

Statistical time series methods for SHM may be classif ed as *excitation-response* or *response-only* methods, depending on whether the characteristic quantity Q is constructed by using or not using, respectively, the excitation signal(s). As already mentioned, they may be also classif ed as *scalar* or *vector*, and as *non-parametric* or *parametric*.

4 OVERVIEW OF STATISTICAL TIME SERIES METHODS FOR VIBRATION–BASED SHM

A concise overview of some of the main methods, both scalar and vector, is provided below.

4.1 Scalar methods

Two non–parametric methods, namely a Power Spectral Density (PSD) and a Frequency Response Function (FRF) based method, and a parametric method, namely a residual variance based method, are brief y reviewed. Their main characteristics are summarized in Table 1.

Power Spectral Density (PSD) based method. Damage detection and identif cation is in this case tackled via changes in the PSD of a measured vibration response signal (non–parametric method). The excitation is assumed unavailable (response–only case). The method's characteristic quantity is the PSD function $Q = S(\omega)$ (ω designating frequency – Table 1). Damage detection is based on conf rmation of statistically signif cant deviations from the nominal healthy state at some frequency [7, 8]. Damage identif cation may be achieved by performing hypothesis tests comparing the current PSD to those corresponding to different damage types and obtained in the baseline phase. It should be noted that response-scaling is important in order to properly account for potentially different excitation levels.

Frequency Response Function (FRF) based method. This is similar to the PSD method, except that it requires the availability of both the excitation and response signals (excitation–response case) and uses the FRF magnitude as its characteristic quantity (non–parametric method), thus $Q = \lvert H(j\omega)\rvert$ with $j = \sqrt{-1}$ (Table 1). The main idea is the comparison of the FRF magnitude of the current structural state to that of the healthy structure. Damage detection is based on conf rmation of statistically signif cant deviations from the nominal healthy state at some frequency [7, 8]. Damage identif cation may be achieved similarly to the previous case.

Residual variance based method. In this method the characteristic quantity is the model residual variance (Table 1). The main idea is based on the fact that the model (which is now parametric) matching the current state of the structure should generate a residual sequence characterized by minimal variance [7, 8].

Thus damage detection may be achieved by examining whether or not the residual variance is minimal [7, 8]. The method uses classical tests on the residuals and offers simplicity and no need for model re–estimation in the inspection phase.

4.2 Vector methods

Two *vector* (multivariate) parametric time series methods for SHM, namely a model parameter based method and a residual likelihood function based method, are brief y reviewed. The main characteristics of the methods are also summarized in Table 1.

Model parameter based method. This method bases damage detection and identif cation on a characteristic quantity Q which is a function of the parameter vector $\boldsymbol{\theta}$ of a parametric time series model (parametric method – see Table 1) [7, 8]. The model has to be re-estimated in the inspection phase based on signals from the current (unknown) state of the structure. Damage detection is based on testing for statistically signif cant changes in the parameter vector $\boldsymbol{\theta}$ between the nominal and current structures through a hypothesis testing problem. Damage identif cation may be based on multiple hypothesis testing comparing the current parameter vector to those corresponding to different damage types. In this article a procedure that uses successive binary tests is employed.

Residual likelihood function based method. In this method damage detection is based on the likelihood function evaluated for the current signal(s) under each one of the considered structural states [25, pp. 119–120], [7, 8]. The hypothesis corresponding to the largest likelihood is selected as true for the current structural state. Damage identif cation is achieved by computing the likelihood function of the current signal(s) for the baseline models corresponding to damaged structural states and accepting the hypothesis that corresponds to the maximum value of the likelihood. By including the healthy baseline model, damage detection is also treated. The method offers simplicity as there is no need for model re–estimation in the inspection phase.

5 THE STRUCTURE AND THE EXPERIMENTAL SET–UP

5.1 The structure

The structure used in the study is a scale aircraft skeleton designed by ONERA in conjunction with the GARTEUR SM-AG19 Group and manufactured at the University of Patras (Fig. 1) [8, 26]. It represents a typical design and consists of six solid beams with rectangular cross sections representing the fuselage ($1500 \times 150 \times 50$ mm), the wing ($2000 \times 100 \times 10$ mm), the horizontal ($300 \times 100 \times 10$ mm) and vertical stabilizers ($400 \times 100 \times 10$ mm), and the right and left wing–tips ($400 \times 100 \times 10$ mm). All parts are constructed from standard aluminum and are jointed together via steel plates and bolts. The total mass of the structure is approximately 50 kg.

5.2 The Damage Types and the Experiments

The structure is suspended through a set of bungee cords and hooks from a long rigid beam sustained by two heavy–type stands (Fig. 1). The suspension is designed in a way to exhibit a pendulum rigid body mode below the frequency range of interest, as the boundary conditions are free–free.

The excitation is broadband random stationary Gaussian applied vertically at the right wing–tip (Point X, Fig. 1) through an electromechanical shaker (MB Dynamics Modal 50A, max load 225 N). The actual force exerted on the structure is measured via an impedance head (PCB M288D01), while the resulting vertical acceleration responses at Points Y1, Y2, Y3 and Y4 (Fig. 1) are measured via lightweight accelerometers (PCB 352A10 ICP). The force and acceleration signals are driven through a conditioning charge amplif er (PCB 481A02) into the data acquisition system based on SigLab 20–42 measurement modules.

The damage considered corresponds to the loosening of a variable number of bolts at different joints of the structure (Fig. 1 – also see [26]). Six distinct types are considered and summarized in Table 2.

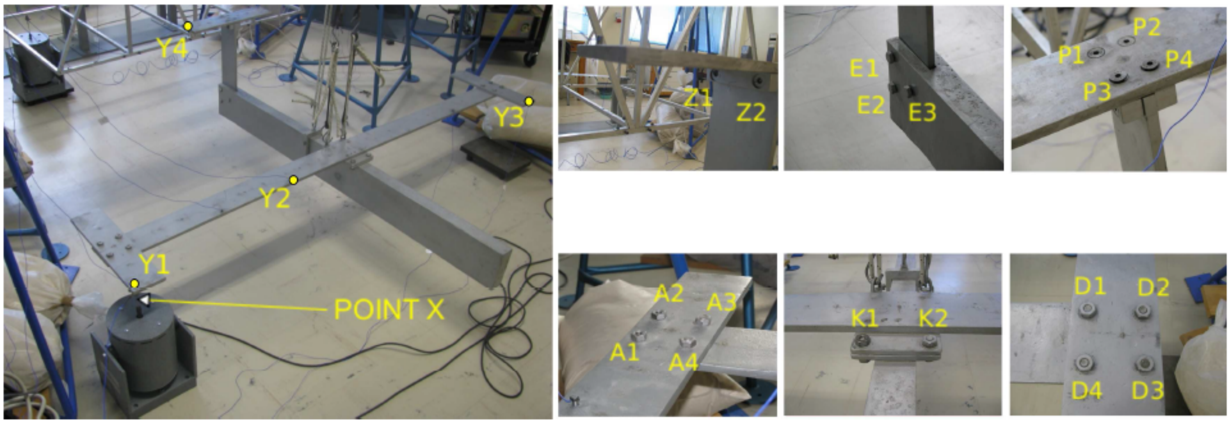

Figure 1: **The aircraft scale skeleton structure and the experimental set-up: The force excitation (Point X), the vibration measurement locations (Points Y1 – Y4), and the bolts connecting the various elements of the structure.**

Table 2: **Experimental details & the damage types**

Structural State	Description	No of Experiments
Healthy	—	60
Damage A	loosening of bolts A1, A4, Z1, Z2	40
Damage B	loosening of bolts D1, D2, D3	40
Damage C	loosening of bolts K1	40
Damage D	loosening of bolts D2, D3	40
Damage E	loosening of bolts D3	40
Damage F	loosening of bolts K1, K2	40
Sampling frequency: $f_s = 512$ Hz, Signal bandwidth: $[4 - 200]$ Hz		
Signal length N in samples (in seconds):		
Non-parametric methods: $N = 46\ 080$ (90 s)		
Parametric methods: $N = 15\ 000$ (29 s)		

The assessment of the statistical time series methods employed is based on 60 experiments for the healthy and 40 experiments for each considered damaged state (damage types A, B, ..., F – see Table 2). Four vibration measurement locations (Fig. 1, Points Y1 – Y4) are employed in order to determine the ability of the methods in treating damage diagnosis using single and multiple vibration response signals.

For damage detection a single healthy data set is used for establishing the baseline (reference) set, while 60 healthy and 240 damaged sets (six damage types with 40 experiments each) are used as inspection data sets. For the damage identif cation task, a single data set for each damaged structural state (damage types A, B,..., F) is used for establishing the baseline (reference) set, while 240 sets are considered as inspection data sets (corresponding to unknown structural states). The time series models are estimated and the corresponding estimates of the characteristic quantity Q are extracted ($\widehat{Q}_A, \widehat{Q}_B, \ldots, \widehat{Q}_F$ in the baseline phase; $\widehat{Q}_u$ in the inspection phase). Damage identif cation is based on successive binary hypothesis tests – as opposed to multiple hypothesis tests [8].

6 EXPERIMENTAL DAMAGE DIAGNOSIS RESULTS

Power Spectral Density (PSD) Based Method. Estimation is based on the Welch Power Spectral Density estimation method, with no–overalp, Hamming window, signals that are $N = 46080$ samples (≈ 90 s)

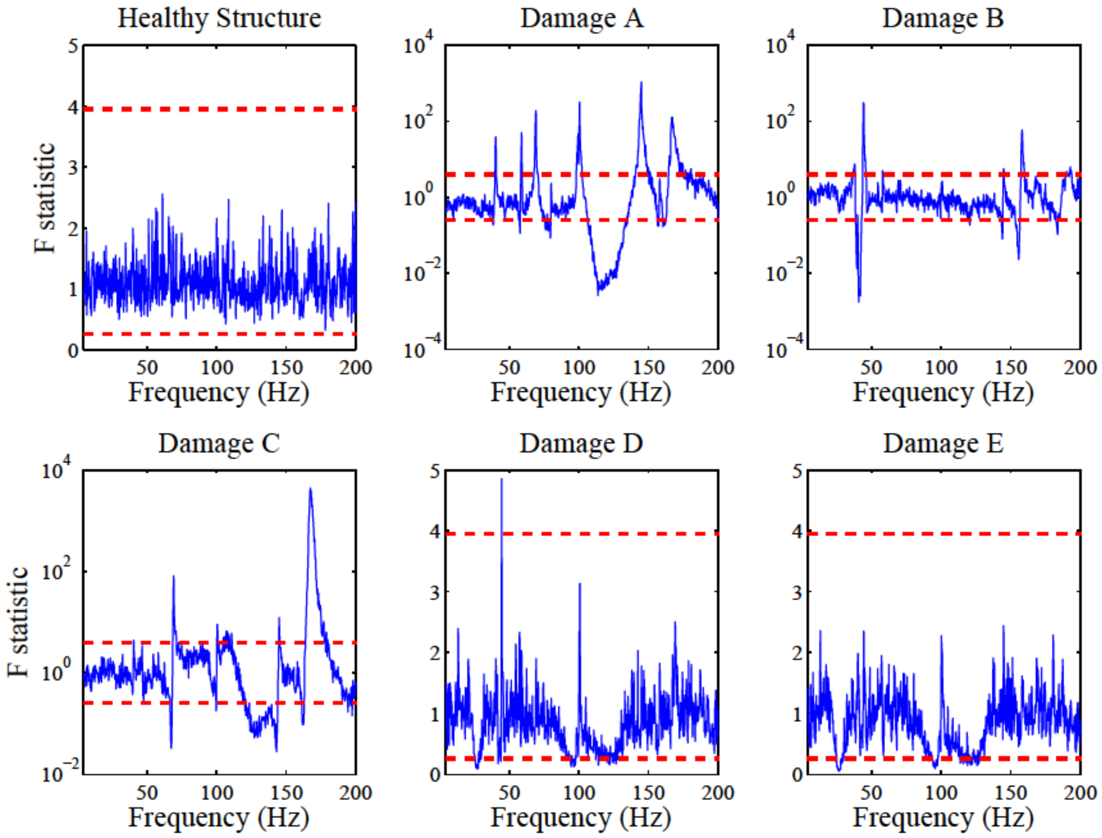

Figure 2: **PSD based method: Indicative damage detection results (output 3) at the $\alpha = 10^{-5}$ risk level. The actual structural state is shown above each plot.**

long, segment length $L = 2048$ samples, and $K = 22$ non–overlapping segments – the achieved frequency resolution is $\delta f = 0.25$ Hz.

Typical damage detection results obtained from the vibration measurement location Y3 (output 3) are presented in Fig. 2. Evidently, correct detection at the $\alpha = 10^{-5}$ risk level is obtained in each case, as the test statistic does not exceed the critical points (dashed horizontal lines) in the healthy case, while it exceeds them in each damage case. It should be observed that damage types D and E are harder to detect.

Summary damage diagnosis results for the considered vibration measurement locations (Fig. 1) are presented in Table 3. The PSD based method achieves accurate damage detection as no false alarms are exhibited, while the number of missed damage cases is zero for all considered damage states. The method is also capable of identifying the actual damage type, as zero damage misclassif cation errors are obtained for damage types A, C, D and F, while some errors errors are reported for damage type E. The misclassif cation errors increase for damage type B when the measurement location Y3 or Y4 are used.

Frequency Response Function (FRF) Based Method. Figure 3 presents damage detection results via the FRF based method obtained at vibration measurement location Y2 (output 2). Evidently, correct detection at the $\alpha = 10^{-6}$ risk level is achieved in each case, as the test statistic is shown not to exceed the critical points (dashed horizontal lines) in the healthy case, while it exceeds the critical point in the damaged cases. Like before, damage types D and E are harder to detect.

Summary damage detection and identif cation results for the considered vibration measurement locations (Fig. 1) are presented in Table 3. The FRF magnitude based method achieves effective damage detection as no false alarms or missed damages are reported (Table 3). The method on the other hand, exhibits decreased accuracy in damage identif cation as signif cant numbers of damage misclassif cation errors are reported for

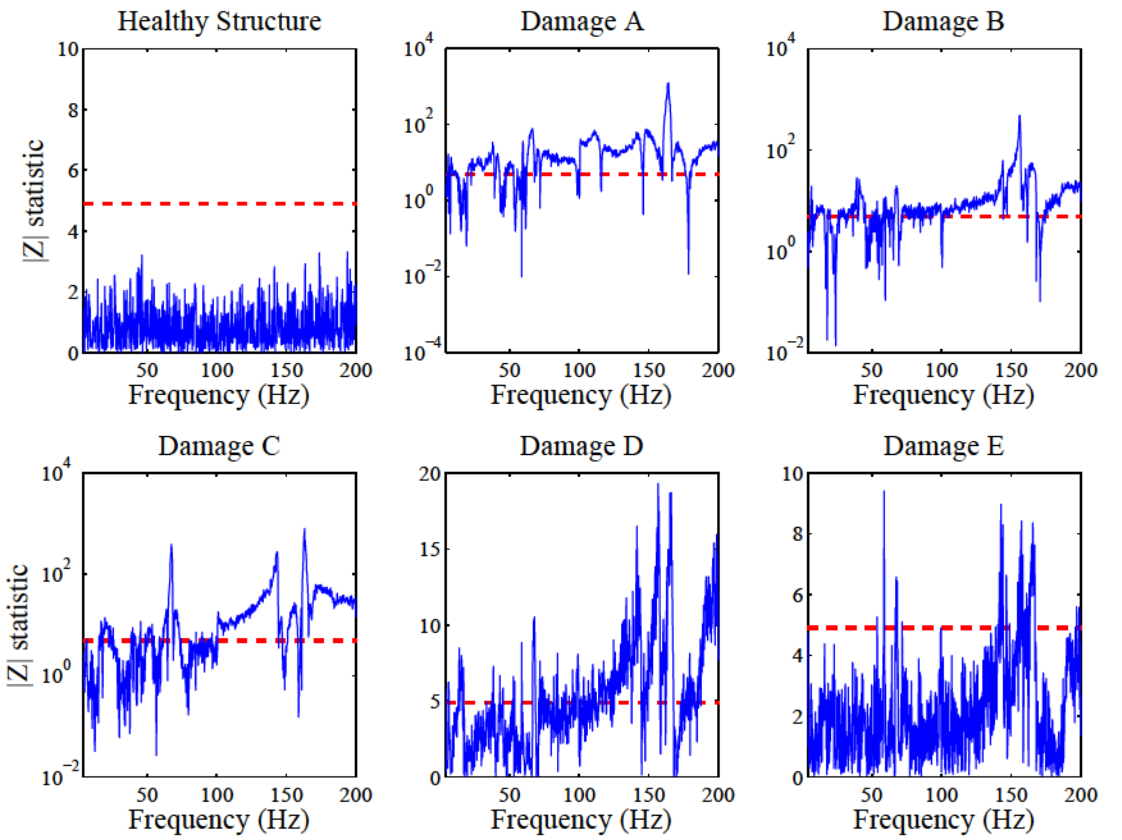

Figure 3: **FRF magnitude based method: Indicative damage detection results (output 2) at the $\alpha = 10^{-6}$ risk level. The actual structural state is shown above each plot.**

damage types B and D.

Residual Variance Based Method. The method is based on identified 4−variate VARX(80, 80) models obtained in the baseline phase, as well as on corresponding models from the current (unknown) data records (inspection phase). Damage detection and identification are achieved via statistical comparison of the two residual variances (each one of the scalar responses is considered separately).

Typical damage detection results obtained via the residual variance based method for vibration measurement location Y2 are shown in Fig. 4. Evidently, correct detection (Fig. 4) is obtained in each considered case, as the test statistic is shown not to exceed the critical point in the healthy case, while it exceeds it in the damaged test cases.

Summary damage detection and identification results for the considered vibration measurement locations are presented in Table 3. The method achieves effective damage detection and identification as no false alarms, missed damages, or damage misclassification cases are observed.

Model parameter Based Method. The model parameter based method (excitation–response case) employs the identified in the baseline phase 4−variate VARX(80, 80) models, as well as an identified VARX(80, 80) model for each current data record (inspection phase).

Figure 5 presents typical parametric damage detection results. The healthy test statistics are shown in circles (60 experiments), while the least severe damage types D and E are presented with asterisks and diamonds, respectively (one for each one of the 40 experiments). Evidently, correct detection is obtained in each case, as the test statistic is shown not to exceed the critical point in the healthy cases, while it exceeds it in the damaged cases (note the logarithmic scale on the vertical axis).

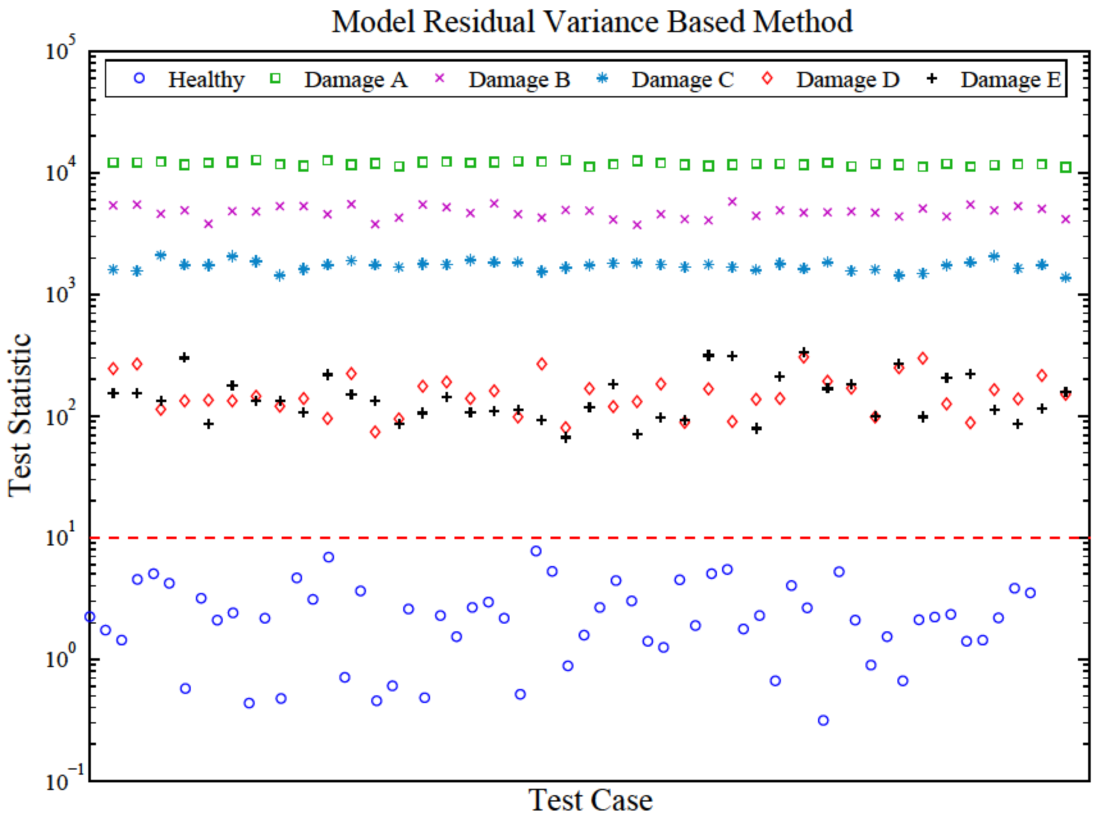

Figure 4: **Residual variance based method: Indicative damage detection results (output 2; healthy – 60 experiments; damaged – 200 experiments). A damage is detected if the test statistic exceeds the critical point (dashed horizontal line).**

As indicated in Table 4, the model parameter based method achieves accurate damage detection and identification, as no false alarm, missed damage, or damage misclassification cases are reported.

Likelihood Function Based Method. The residual likelihood function based method is based on the identified 4–variate VARX(80, 80) models from the baseline phase. Figure 6 presents typical damage detection results. Correct detection is obtained in each case, as the test statistic is shown not to exceed the critical point in the healthy cases, while it exceeds it in the damaged cases.

The method achieves accurate damage detection and identification, as no false alarm, missed damage, or damage misclassification cases are reported. Summary damage detection and identification results are presented in Table 4.

6.1 Discussion

Scalar time series methods for SHM are shown to achieve effective damage diagnosis, although non–parametric scalar methods encounter some difficulties. The PSD based method achieves excellent damage diagnosis, although it exhibits some misclassification errors for damage type E. The misclassification errors increase for damage type B and the Y3 and Y4 vibration measurement locations. The FRF based method achieves accurate damage detection with no false alarms or missed damages, except for vibration measurement location Y4 for which it exhibits an increased number of false alarms. Moreover, it faces problems in correctly identifying damage types B and D, as the number of damage misclassification cases is higher for these damage types which involve loosening of bolts on the left wing–tip of the aircraft (Fig. 1). On the other hand, the parametric residual variance based method achieves excellent performance in accurately detecting

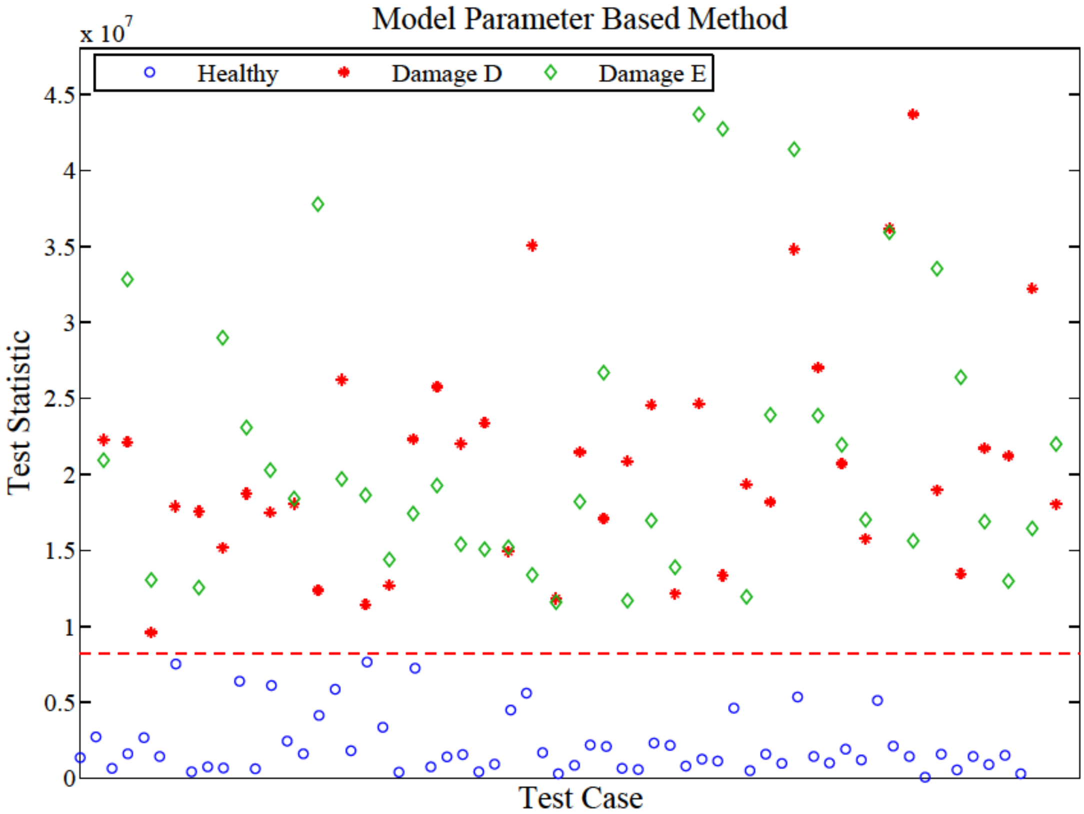

Figure 5: **Model parameter based method: Indicative damage detection results for three structural states (healthy – 60 experiments; damaged – 80 experiments). A damage is detected if the test statistic exceeds the critical point (dashed horizontal line).**

and identifying damage for all considered vibration measurement locations.

Vector time series methods for SHM achieve very accurate damage diagnosis, as with properly adjusted risk level α (type I error) no false alarm, missed damage, or damage misclassif cation cases are observed. Moreover, the methods demonstrate global damage detection capability. Nevertheless, parametric vector models require accurate parameter estimation and appropriate model structure (order) selection in order to accurately represent the structural dynamics and effectively tackle damage diagnosis. Therefore, these methods require user expertise and are somewhat more elaborate than their scalar or non–parametric counterparts.

Furthermore, the number and location of vibration measurement sensors is an important issue. Several vibration based damage diagnosis techniques that appear to work well in certain cases, could actually perform poorly when subjected to the measurement constraints imposed by actual testing [2]. Techniques that are to be seriously considered for implementation in the f eld should demonstrate that they can perform well under limitations such as a small number of measurement locations and the constraint that these locations should be selected a–priori, without knowledge of the actual damage location.

It is also noteworthy that in order for certain parametric methods to work effectively, a very small value of the type I risk α is often needed. This is due to the fact that the currently used stochastic time series models do not fully capture the experimental, operational and environmental uncertainties that the structure is subjected to.

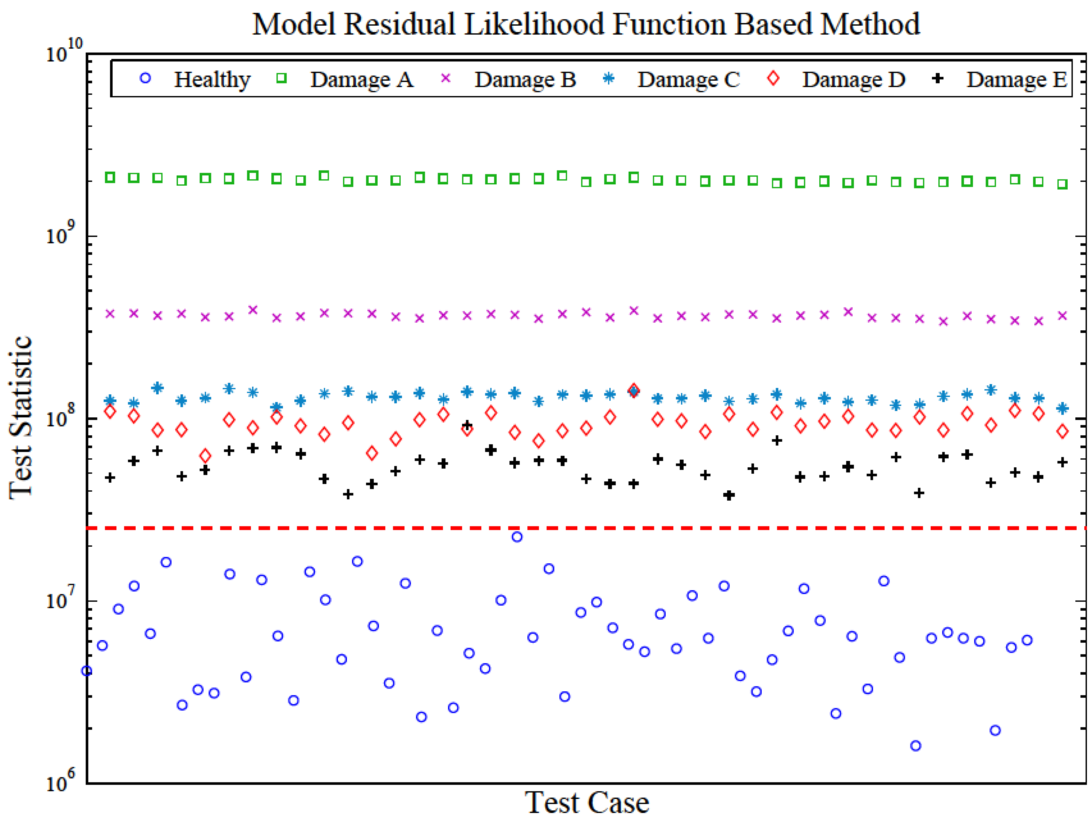

Figure 6: **Residual likelihood function based method: Indicative damage detection results (healthy – 60 experiments; damaged – 200 experiments). A damage is detected if the test statistic exceeds the critical point (dashed horizontal line).**

Table 3: **Scalar methods damage detection & identif cation – summary results**

Method	Damage Detection							Damage Identif cation					
	False alarms	Missed damage						Damage misclassif cation					
		dam. A	dam. B	dam. C	dam. D	dam. E	dam. F	dam. A	dam. B	dam. C	dam. D	dam. E	dam. F
PSD based													
response Y1	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
response Y2	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
response Y3	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	**21/40**	0/40	0/40	1/40	0/40
response Y4	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	**21/40**	0/40	0/40	2/40	0/40
FRF based													
response Y1	1/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	**10/40**	**6/40**	**5/40**	2/40	0/40
response Y2	0/60	0/40	0/40	0/40	0/40	1/40	0/40	0/40	**4/40**	**10/40**	**22/40**	**9/40**	3/40
response Y3	0/60	0/40	0/40	0/40	1/40	0/40	0/40	0/40	**7/40**	2/40	**9/40**	**5/40**	1/40
response Y4	**35/60**	0/40	0/40	0/40	0/40	0/40	0/40	0/40	**8/40**	0/40	**8/40**	2/40	0/40
Res. variance†													
response Y1	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
response Y2	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
response Y3	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
response Y4	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40

† adjusted α

7 CONCLUDING REMARKS

- Statistical time series methods for vibration–based SHM achieve effective damage detection, identif cation (including localization), and damage magnitude estimation based on (i) random excitation and/or vibration responses, (ii) statistical model building, and (iii) statistical decision making under uncertainty.

Table 4: **Vector methods damage detection and identif cation – summary results**

Method	Damage Detection							Damage Identif cation					
	False	Missed damage						Damage misclassif cation					
	alarms	dam. A	dam. B	dam. C	dam. D	dam. E	dam. F	dam. A	dam. B	dam. C	dam. D	dam. E	dam. F
Mod. parameter†	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40
Res. likelihood†	0/60	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40	0/40

† adjusted α

- The methods are data–based, inverse–type, and of general applicability.
- In addition to sharing the benef ts of general vibration based methods (like “global” coverage, time and cost effectiveness, automation capability) statistical time series methods offer a number of unique advantages: (i) No need for physics–based or f nite element models; (ii) no need for complete structural models (partial models and a limited number of responses suff ce); (iii) inherent accounting of uncertainty; (iv) statistical decision making with specif ed performance characteristics; (v) potential use of ambient random vibration data records.
- The methods’ ability to provide effective damage diagnosis using a low–frequency bandwidth and a very small number of sensors is remarkable. It is also very important, as in practice the excitation may be ambient (and thus of limited bandwidth), while the number of sensors may need to be (due to various reasons) constrained.
- Statistical time series methods may be either of the non–parametric or parametric types. The latter are generally more elaborate, but offer potentially improved capabilities.
- The methods offer “global” damage diagnosis, as they are able to diagnose damage that is either “local” or “remote” with respect to the sensor location used.
- The availability of data records corresponding to various potential damage scenarios is necessary for damage identif cation and magnitude estimation. This may not be possible with the actual structure itself, but laboratory scale models or analytical (like Finite Element) models may be used for this purpose in the baseline (training) phase.

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